A New Reciprocity Theorem

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***Abstract*— A new reciprocity theorem for isotropic media is presented. This new theorem accounts for the effect of cross polarization therefore gracefully complementing the accepted, and century old, reciprocity theorem.**

I. Introduction

ONE OF THE BASIC and most important theorems of electromagnetic theory is the so-called reciprocity theorem. Its importance is evident from its wide range of applicability in all branches of electrical engineering. Its history is rather long in view of its many contributors.

In 1877 Lord Rayleigh established a theorem of reciprocity theorem for a linearized vibrating mechanical system [1]. Twenty-three years later he extended this to optics [2]. However, he did not consider phases, just the state of polarization of each beam of light. It appears that Rayleigh's work was first extended to electromagnetic waves by Lorentz [3], which is why the basic sourceless reciprocity theorem for isotropic media, as we know it today, is usually referred to as the Lorentz form [4], [5]. It is important to mention the significantly less-known fact [6], that in early 1877 Oliver Heaviside published a paper related to an underwater electrical cable [7], wherein he invoked what is today known as reciprocity theorem for electrical networks. Heaviside was never credited as being a pioneer in this area, possibly due to the lack of readability of his works.

This early work was soon generalized to vector fields, and later, to include complex anisotropy and time dependence. Notable among the new research is the work of Welch [8] who obtained a reciprocity relation for time-dependent electromagnetic fields in a homogeneous nondispersive medium which involved retarded fields and both electric and magnetic sources. Another important work is that of Kong and Cheng [9], which handled the general time-harmonic problem of full bianisotropic media and departed significantly from the previous line of thought in that the relations obtained involved a so-called complementary region, which was characterized by a material composition different from that of the original space.

The transition in the analysis from isotropic to bianisotropic was not abrupt. There were numerous works on anisotropic media involving different constitutive relations and/or different boundary surface (such as surface impedance) and sometimes inhomogeneities. More recently [10], the anisotropic work has been extended to the case of inhomogeneous piezo-electric media (bounded by a surface impedance), thereby including the dielectric/magnetic and elastic properties of materials. Since the present work is purely isotropic, references to nonisotropic cases will be kept rather brief, and from now on we will concentrate exclusively on the isotropic case.

We should also mention an interesting paper by Tai [11] who by independent means derived a reciprocity relation which bears some resemblance to the concept presented here. However, it involves two sets of fields which satisfy complementary impedance conditions (one of the fields being introduced as not being physically realizable) and is applied exclusively to a layered configuration above a ground plane (for it appears that the goal of the paper was to arrive at a symmetrical relationship of two magnetic dyadic Green's functions).

In reality, when we deal with fields in an isotropic dielectric/magnetic medium, we are really invoking the quantum mechanical problem of emission and absorption of photons by atoms. Thus, reciprocity is intimately related to the well-known principle of equal probabilities for inverse transitions between two states of the same energy. Photons are characterized by spin, which is intimately related to the macroscopic circular polarization states we employ in electrodynamics.

It follows that the proper frame of analysis for the macroscopic problem should involve the two circularly polarized fields, each of which will be characterized by a reciprocity relation. We are talking about general fields and sources and not just plane waves. It is clear that space inversion and time reversal symmetries will constrain these two reciprocity relations, however, we cannot *a priori* conclude that the two contain a single element of information.

That something is missing from the currently accepted reciprocity theorem can be ascertained from the theorem which says that: the field from source A evaluated at and in the direction of source B is equal to the field from source B evaluated at and in the direction of source A. Note that the theorem involves only the fields polarized in the direction of the sources. Nothing is said about the cross-polarized fields. That information is not contained in the accepted form.

A reciprocity theorem, in principle, is expected to account for and specify all mutual exchanges between field distributions. For instance, if we have an electric dipole $\overline{J}_1$, such as shown in Fig. 1, we expect that a suitable theorem should give an indication of the fields produced by $\overline{J}_1$ on any other element such as $\overline{J}_2$ (see Fig. 1). The standard reciprocity theorem [presented here in (23)] cannot account for this exchange and dismisses it in the context of "absence of reaction" between

Manuscript received May 2, 1994; revised October 2, 1995. This work was supported in part by the U.S. Naval Research Laboratories Contract N00014-91-C-2179 with Damaskos, Inc.

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Publisher Item Identifier S 0018-9480(96)00482-6.

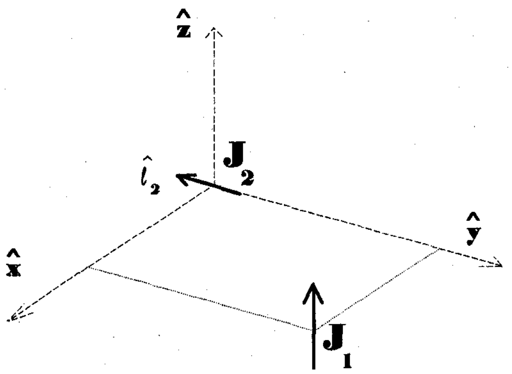

Fig. 1. Related to the coupling between elemental dipoles.

the elements. We know however that there is more to it since J_1 produces an H field along J_2 and vice versa.[1]

In what follows, we demonstrate that there is another basic relation; a missing reciprocity theorem which accounts for the cross-polarized field components and therefore gracefully complements the currently accepted theorem. The practical and analytical relevance of this fundamental theory is evident.

II. Analysis

The initial step in our derivation is a field decomposition into RCP (right circularly polarized or "+" wave) and LCP (left circularly polarized or "−" wave) components. It is well known that in a homogeneous isotropic space, Maxwell equations admit plane wave solutions of the form

$$\overline{E} = \overline{e}e^{-j\overline{v}\cdot\overline{r}}; \qquad \overline{H} = \overline{h}e^{-j\overline{v}\cdot\overline{r}} \tag{2}$$

where v is the wavenumber, and where $\overline{e}$ and $\overline{h}$ are orthogonal to the direction of propagation $\hat{v}$ and given by

$$\overline{e}_\pm = \pm j\eta_\pm \overline{h}_\pm \tag{3}$$

$$\overline{h}_\pm = \frac{1}{\sqrt{2}}(\hat{s} \mp j\hat{t}) \tag{4}$$

for $[\hat{v}\hat{s}\hat{t}]$ forming a right-handed triad. Here we assume a time convention $\exp\{j\omega t\}$ which is suppressed throughout.

Equation (3) indicates that $\overline{e}_\pm$ is proportional to $\overline{h}_\pm$, the constant of proportionality being independent of the direction of propagation $\hat{v}$. Since the total field solution in the presence of sources can be expressed as a summation of (2) over a bundle of inhomogeneous waves; the following decomposition, which includes electric and magnetic sources, becomes apparent

$$\overline{E} = \overline{E}_+ + \overline{E}_-; \qquad \overline{H} = \overline{H}_+ + \overline{H}_- \tag{5}$$

$$\overline{J} = \overline{J}_+ + \overline{J}_-; \qquad \overline{M} = \overline{M}_+ + \overline{M}_-. \tag{6}$$

And the above argument coupled with (3) results in

$$\overline{E}_\pm(\overline{r}) = \mp j\eta\overline{H}_\pm(\overline{r}). \tag{7}$$

The point relationship (7) is a remarkable property of the fields, which is satisfied everywhere, even in the neighborhood of singularities. Similar point relationships have been found to hold even in the more complex biisotropic case [12].

Insertion of (5)–(7) into Maxwell equations

$$\nabla \times \overline{H} = j\omega\epsilon\overline{E} + \overline{J}, \qquad \nabla \times \overline{E} = -j\omega\mu\overline{H} - \overline{M} \tag{8}$$

and after grouping the resulting terms into RCP and LCP components, we find that the separation is legitimate provided the sources are decomposed according to

$$\overline{J}_\pm = \pm\frac{\overline{M}_\pm}{j\eta}; \qquad \overline{M}_\pm = \frac{1}{2}[\overline{M} \pm j\eta\overline{J}] \tag{9}$$

and provided the circularly polarized components satisfy

$$\nabla \times \overline{H}_\pm = \pm v\overline{H}_\pm + \overline{J}_\pm. \tag{10}$$

The inversion of (10) can be done via standard methods, the solution being

$$\overline{H}_\pm = -L_\pm \cdot \overline{\Psi}_\pm \tag{11}$$

$$L_\pm = \nabla \times \overline{\overline{I}} \pm v\overline{\overline{I}} \pm \frac{\nabla\nabla}{v} \tag{12}$$

where $\overline{\Psi}_\pm$ satisfies the simple equation

$$(\nabla^2 + v^2)\overline{\Psi}_\pm = \overline{J}_\pm. \tag{13}$$

Equation (9) gives the conditions for excitation. Single-mode excitation for instance can be achieved via parallel/antiparallel loop ($\overline{M}$) and dipole ($\overline{J}$) of fixed ratio $j\eta$.

Let us now define two sets of sources, ($\overline{J}^{(1)}$, $\overline{M}^{(1)}$) and ($\overline{J}^{(2)}$, $\overline{M}^{(2)}$), which according to (9) result in the two sets of sources $\overline{J}_\pm^{(1,2)}$, with corresponding sets of fields $H_\pm^{(1,2)}$. Equation (10) can be independently applied to each set of sources yielding

$$\nabla \times \overline{H}_\pm^{(1,2)} = \pm v\overline{H}_\pm^{(1,2)} + \overline{J}_\pm^{(1,2)}. \tag{14}$$

If we dot this equation with $\overline{H}_\pm^{(2,1)}$ and subtract the resulting two component equations followed by use of the identity $\overline{B}\cdot\nabla\times\overline{A} - \overline{A}\cdot\nabla\times\overline{B} = \nabla\cdot(\overline{A}\times\overline{B})$, we obtain

$$\nabla \cdot \{\overline{H}_\pm^{(1)} \times \overline{H}_\pm^{(2)}\} \equiv u_\pm \tag{15}$$

where

$$u_\pm = \overline{H}_\pm^{(2)}\cdot\overline{J}_\pm^{(1)} - \overline{H}_\pm^{(1)}\cdot\overline{J}_\pm^{(2)}. \tag{16}$$

Integration of (15) over all space, followed by application of Gauss's theorem, yields the identity

$$\psi_\pm + \int_\infty d\tau u_\pm = 0 \tag{17}$$

where the integral sign denotes volumetric integration over infinite space, and where ψ is given by

$$\psi_\pm = \oint_{S_\infty} dS\,\hat{n}\cdot\{\overline{H}_\pm^{(1)} \times \overline{H}_\pm^{(2)}\} \tag{18a}$$

for $\hat{n}$ the outward-pointing unit vector normal to the surface S_∞. Via (5) and (7), the above integral can be rewritten in

[1] The reader is cautioned not to envision the elementary sources as wires for in that case this point may appear trivial. On the other extreme we have the case of a "source" being a magnetoelectric rod where the incident axial magnetic field induces an axial electric (displacement) current.

terms of the total fields as

$$\psi_{\pm} = \frac{1}{4}\oint_{S_\infty} dS\,\hat{n}\cdot\left\{\left[\overline{H}^{(1)}\times\overline{H}^{(2)} - \frac{1}{\eta^2}\overline{E}^{(1)}\times\overline{E}^{(2)}\right] \pm \frac{j}{\eta}[\overline{H}^{(1)}\times\overline{E}^{(2)} + \overline{E}^{(1)}\times\overline{H}^{(2)}]\right\}. \quad (18b)$$

The integral of the second square bracket is zero in view of the Lorentz reciprocity theorem [4], [5]. That the whole integral (18b) is zero can be easily shown from the fact that at an infinite distance away from all sources the $\overline{E}$ and $\overline{H}$ fields are related by $\overline{H} = \hat{n}\times\overline{E}/\eta$. Thus we have that in general, with no constraint in infinite homogeneous space

$$\int_\infty d\tau u_{\pm} = 0. \quad (19)$$

Note that we could have gone from (15) to (19) directly via Gauss's theorem and the assumption of very small material losses; that is however just a convenience, somewhat restrictive and unnecessary.

The above expression (19) is actually a strong statement, for it implies that reciprocity is obeyed by each characteristic mode, independently. Note that (19) consists of two equations. In what follows we prove that those equations are independent.

To express (19) in terms of total fields, we express the partial fields in terms of the total fields via (5) and (7)

$$\overline{H}_{\pm} = \frac{1}{2}\left[\overline{H} \pm j\frac{\overline{E}}{\eta}\right]. \quad (20)$$

Use of this and (9) in the definition of $u_{\pm}$ given by (16), results in

$$u_{\pm} = \frac{1}{2j\eta}\left\{\begin{array}{l} \pm\eta[\overline{H}^{(2)}\cdot\overline{M}^{(1)} - \overline{H}^{(1)}\cdot\overline{M}^{(2)}] \\ \quad +j\eta^2[\overline{H}^{(2)}\cdot\overline{J}^{(1)} - \overline{H}^{(1)}\cdot\overline{J}^{(2)}] \\ +j[\overline{E}^{(2)}\cdot M^{(1)} - \overline{E}^{(1)}\cdot\overline{M}^{(2)}] \\ \quad \mp\eta[\overline{E}^{(2)}\cdot\overline{J}^{(1)} - \overline{E}^{(1)}\cdot\overline{J}^{(2)}] \end{array}\right\}. \quad (21)$$

For reasons that will become clear very shortly, instead of directly enforcing (19), we will enforce the two equivalent expressions

$$\int_\infty d\tau(u_+ \pm u_-) = 0. \quad (22)$$

This results in the following two identities:

1) The difference [lower sign in (22)]

$$\int_\infty d\tau(\overline{J}^{(1)}\cdot\overline{E}^{(2)} - \overline{M}^{(1)}\cdot\overline{H}^{(2)}) = \int_\infty d\tau(\overline{J}^{(2)}\cdot\overline{E}^{(1)} - \overline{M}^{(2)}\cdot\overline{H}^{(1)}) \quad (23)$$

which is the standard reciprocity theorem and forms the basis for the definition of reactions, which in turn has resulted in a general procedure for the establishment of stationary formulas [13]. Impressive developments in microwave theory as we know it, have been the product of wise exploitation of (23). Standard reciprocity (23) is usually abbreviated as $\langle 1, 2\rangle = \langle 2, 1\rangle$.

2) The addition [upper sign in (22)]

$$\int_\infty d\tau(\overline{M}^{(1)}\cdot\overline{E}^{(2)} + \eta^2\overline{J}^{(1)}\cdot\overline{H}^{(2)}) = \int_\infty d\tau(\overline{M}^{(2)}\cdot\overline{E}^{(1)} + \eta^2\overline{J}^{(2)}\cdot\overline{H}^{(1)}) \quad (24)$$

which constitutes a new reciprocity theorem. It is not related to the standard theorem (23), even though it bears some resemblance when one considers one set of sources to exist in dual space ($\epsilon \leftrightarrow \mu$, $\overline{E}\to\overline{H}$, $\overline{H}\to -\overline{E}$, PEC $\to$ PMC, etc.). This is not the case though, since all sources in (24) cohabit the same space.

Another demonstration that (23) and (24) are independent arises when one tries to extend (24) to regions V of finite extent bounded by perfect conductors (electric or magnetic). When this happens, $\psi_{\pm}$ is no longer zero, and (23) and (24) should be augmented by the difference and the sum respectively of the two elements (integrals) presented in (18b). For S the finite surface enclosing V, (23) should be augmented by

$$\frac{j}{2\eta}\oint_S dS\,\hat{n}\cdot[\overline{H}^{(1)}\times\overline{E}^{(2)} + \overline{E}^{(1)}\times\overline{H}^{(2)}]. \quad (25)$$

The above integral being zero because on a perfect conductor either $\overline{E}\times\hat{n}$ is zero (electric, PEC), or $\hat{n}\times\overline{H}$ is zero (magnetic, PMC). On the other hand, (24) should be augmented by

$$\frac{1}{2}\oint_S dS\,\hat{n}\cdot\left[\overline{H}^{(1)}\times\overline{H}^{(2)} - \frac{1}{\eta^2}\overline{E}^{(1)}\times\overline{E}^{(2)}\right] \quad (26)$$

which is clearly nonzero for perfect conductors in general. This example clearly shows that (23) and (24) are completely independent because the former applies to finite regions and the latter needs an extra term.

Note that (24) is directly applicable to the geometry of Fig. 1, where it results in a trivial statement of symmetry. If we complicate the situation slightly, such as shown in Fig. 2, via an axial electric line source and a line source of transversely-directed current elements, we end up with the less trivial result that the axial field H_z due to the transversal currents is equal to the projection of the magnetic field from the axial source, in the direction of the transversal currents. Notice that the standard theorem (23) is still not applicable.

It should also be mentioned, for the sake of completeness, that (24) bears no relation to the modified reciprocity theorem [9], which applies to bianisotropic media in general, but which does not depart from (23) for the isotropic case we treat here.

Since reciprocity in the sense of (24) is different to reciprocity in the sense of (23), for convenience we adopt

$$[a, b] = \int_\infty d\tau\{\eta^2\overline{J}^{(a)}\cdot\overline{H}^{(b)} + \overline{M}^{(a)}\cdot\overline{E}^{(b)}\}. \quad (27)$$

With this abbreviation, the new reciprocity theorem can be compactly written as

$$[a, b] = [b, a]. \quad (28)$$

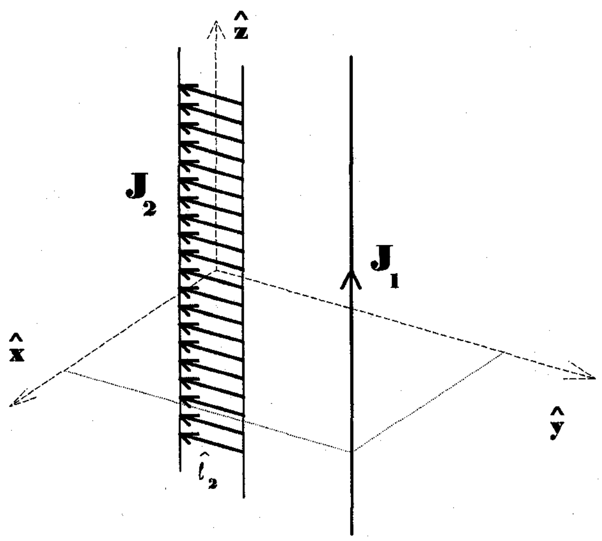

Fig. 2. Related to the coupling between line sources.

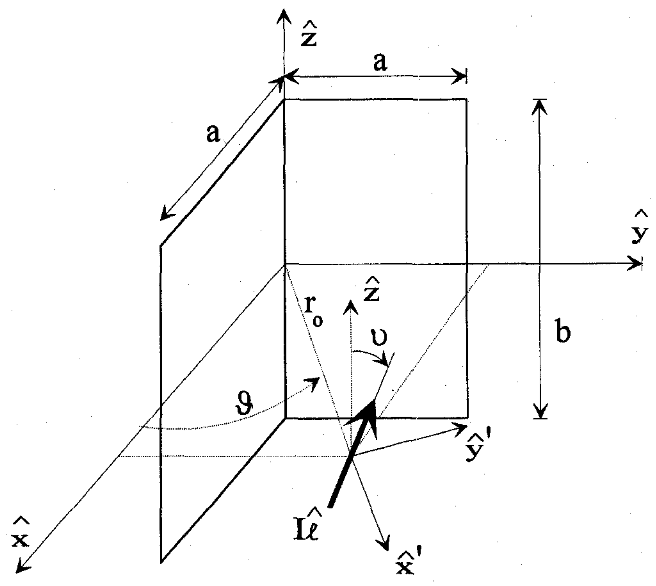

Fig. 3. The dihedral corner reflector as a depolarizing structure.

III. Further Discussion and Elementary Applications

It may appear at first that the usefulness of (27) and (28) is hampered by the need to deal with an infinite homogeneous space. This is not so, and has been truly demonstrated by all the clever developments used to exploit the standard theorem (23). A simple way through which (27) and (28) can be applied to general regions is by replacing the inhomogeneities and scatterers by equivalent surface currents (electric and magnetic) or via displacement (volumetric) currents (electric and magnetic), all of which are made to radiate in free space.

In order to appreciate the power of the new reciprocity theorem, we consider the case of scattering by an arbitrarily shaped perfect electric conductor of surface S. We will use an electric dipole source $\overline{J}^a = I_o \hat{l}\, \delta(\overline{x} - \overline{x}_o)$, which produces fields $\overline{H}^{\mathrm{inc}}(\overline{x}, \overline{x}_o)$ and $\overline{E}^{\mathrm{inc}}(\overline{x}, \overline{x}_o)$ at point $\overline{x}$.

Under the circumstances we replace the conductor by the electric surface currents $\overline{J}_s(\overline{x})$, which radiate in free space. There will be no magnetic currents, and the volumetric integrals in (23) and (24) become surface integrals over S. The new reciprocity theorem results in the scattered magnetic field

$$\overline{H}^s(\overline{x}_o) \cdot \hat{l} = \frac{1}{I_o} \oint_S dS\, \overline{J}_s(\overline{x}) \cdot \overline{H}^{\mathrm{inc}}(\overline{x}, \overline{x}_o) \qquad (29)$$

whereas the standard form yields the scattered electric field

$$\overline{E}^s(\overline{x}_o) \cdot \hat{l} = \frac{1}{I_o} \oint_S dS\, \overline{J}_s(\overline{x}) \cdot \overline{E}^{\mathrm{inc}}(\overline{x}, \overline{x}_o). \qquad (30)$$

Equation (30) expresses the copolarized component of the scattered field, is well known, and has been used as the basis for stationary formulas for monostatic scattering. Equation (29) on the other hand appears to be new and relates essentially to the crosspolarized scattered component. A sound conclusion can be made: That the new reciprocity theorem complements the old one in that the new one is eminently cross-polarized while the old one is eminently copolarized.

In passing, we note that (29) can be used as the starting point for a development on variational expressions for cross-polarization involving metallic bodies. It can also be used (via plane wave incidence) as a new formula for the cross-polarized scattering cross section. Most importantly, note that by redefining $\overline{J}_s(\overline{x})$ as due to an independent source and suitably choosing the observation point we may obtain new integral equations for scattering. These and other avenues will be explored in a forthcoming paper.

It appears that the new theorem is ideal for complex structures or simple structures with poor symmetry such that they exhibit a large degree of crosspolarization. For instance, an arbitrarily shaped thin wire scatterer (e.g., a spiral) cannot be analyzed via the old theorem alone; use must be made of (27), (28) for the important crosspolarized components.

A very interesting example of the need of the new theorem is afforded by the important problem of normal incidence on an ideal dihedral corner (see Fig. 3), for which it is known that the scattered field is depolarized except for polarization either along the axes or perpendicular to it. When the incident polarization makes 45° with the axes, the scattered field is entirely crosspolarized (thereby rendering the classical reciprocity theorem useless). A simplified application of (29), (30) to the corner reflector is included in the Appendix.

To end this presentation we resort to a dramatic feature of (29). If we employ physical optics (PO), namely $\overline{J}_s = 2\hat{n} \times \overline{H}^{\mathrm{inc}}$, we immediately see from (29) that

$$\overline{H}^s_{PO}(\overline{x}_o) \cdot \hat{l} = 0. \qquad (31)$$

It follows that (29) displays the fact that the PO approximation does not depolarize under monostatic conditions (regardless of separation between source and scatterer). This is a well-known result which yields more confidence on (27), (28).

IV. Conclusion

A new reciprocity relation has been obtained via the use of a general field expansion in terms of circularly polarized components. This new theorem, unlike the currently accepted reciprocity theorem, accounts for crosspolarization, and therefore gracefully complements the accepted form. The new theorem is compactly written as $[a, b] = [b, a]$ as opposed to the traditional theorem which is abbreviated as $\langle a, b\rangle = \langle b, a\rangle$. The validity and significance of the new theorem is discussed and elementary applications included.

APPENDIX

Here we apply (29) and (30) to an electrically large dihedral corner reflector which is illustrated in Fig. 3. The source is a distant electric dipole of strength Il located in the plane $z = 0$ and at a distance r_o, which produces a field of the form

$$\overline{E}^{\text{inc}} = C\hat{l}\, e^{j(\alpha x+\beta y)}; \qquad \overline{H}^{\text{inc}} = \frac{1}{\eta}\overline{E}^{\text{inc}} \times \hat{x}' \tag{A1}$$

where

$$\alpha = k\,\text{Cos}\,\theta, \qquad \beta = k\,\text{Sin}\,\theta \tag{A2}$$

and

$$C = j\eta Ilk^2 \frac{e^{-jkr_o}}{4\pi kr_o}. \tag{A3}$$

The direction $\hat{l}$ forms an angle v with the $\hat{z}$ axis. The incident field is locally a plane wave which propagates at an angle θ with respect to the $\hat{x}$ axis.

Equations (29) and (30) require an exact expression for the current induced on the reflector. No exact analytical solution exists, however, we will ignore diffraction by the edges and approximate the current via geometrical optics (physical optics is not appropriate here due to the multiple reflections). In the neighborhood of the reflector surface the fields can be approximated by the effect of the source dipole plus its three images (obtained by standard means, as if the reflector was infinite in extent). As usual the current is given by $\overline{J} = \hat{n} \times \overline{H}$. After some algebra we obtain:

1) In the x-z plane $(0 \le x \le a,\ -b/2 \le z \le b/2)$

$$\overline{J} \approx -\frac{4}{\eta}C \cdot \{\text{Cos}\,(ax) \cdot \text{Sin}\,v\hat{x} - j\,\text{Sin}\,\theta \cdot \text{Cos}\,v \cdot \text{Sin}\,(\alpha x)\hat{z}\}. \tag{A4}$$

2) In the y-z plane $(0 \le y \le a,\ -b/2 \le z \le b/2)$

$$\overline{J} \approx \frac{4}{\eta}C \cdot \{\text{Cos}\,(\beta y) \cdot \text{Sin}\,v\hat{y} + j\,\text{Cos}\,\theta \cdot \text{Cos}\,v \cdot \text{Sin}\,(\beta y)\hat{z}\}. \tag{A5}$$

Next we evaluate (29)–(30), retaining dominant terms and neglecting terms of order (1/ka). We obtain $\overline{H}^s \cdot \hat{l}$ and $\overline{E}^s \cdot \hat{l}$ which can be used to define the cross-polarized cross section σ_{cross} and copolarized cross section σ_{co}, respectively

$$\sigma_{\text{cross}} = 4\pi r_o^2 \left|\frac{\overline{H}^s \cdot \hat{l}}{\overline{H}^{\text{inc}}}\right|^2; \qquad \sigma_{\text{co}} = 4\pi r_o^2 \left|\frac{\overline{E}^s \cdot \hat{l}}{\overline{E}^{\text{inc}}}\right|^2. \tag{A6}$$

After some lengthy algebra we obtain

$$\sigma_{\text{cross}} = \frac{8\pi a^2 b^2}{\lambda^2}\,\text{Sin}^2\,(\theta + \pi/4) \cdot \text{Sin}^2\,(2v) \tag{A7}$$

$$\sigma_{\text{co}} = \frac{8\pi a^2 b^2}{\lambda^2}\,\text{Sin}^2\,(\theta + \pi/4) \cdot \text{Cos}^2(2v). \tag{A8}$$

When $v = 0$ or $\pi/2$ (i.e., polarization along the axes or perpendicular to it), there is no depolarization, $\sigma_{\text{cross}} = 0$ and σ_{co} assumes the usual expression for a reflector [14]–[15]. Aside from this condition, the standard reciprocity theorem (as measured by σ_{co}) does not reflect the true coupling between sources (dipole and currents on the reflector). When $v = \pi/4$ for instance σ_{co} does not predict coupling, whereas σ_{cross} is a maximum indicating complete crosspolarization. The new reciprocity theorem clearly complements the old one.

ACKNOWLEDGMENT

The author would like to thank an anonymous reviewer who was able to rederive one of the expressions presented here by independent and simpler means.

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